Physical Modelling of Musical Instruments Using Digital Waveguides:

History, Theory, Practice

Introduction

- Why Physical Modelling?
- History of Waveguide Physical Models
- Mathematics of Waveguide Physical Models, via Data Flow Diagrams
- Demonstration of Yamaha VL synthesizer
- Why Physical Modelling?

What Physical Modelling is NOT

- NOT looped playback of previously recorded sound (e.g. samplers)

What Physical Modelling is NOT

- NOT looped playback of previously recorded sound (e.g. samplers)

What Physical Modelling is NOT

- NOT synthesis by *ad hoc* similarity of sound (e.g. subtractive synthesis)

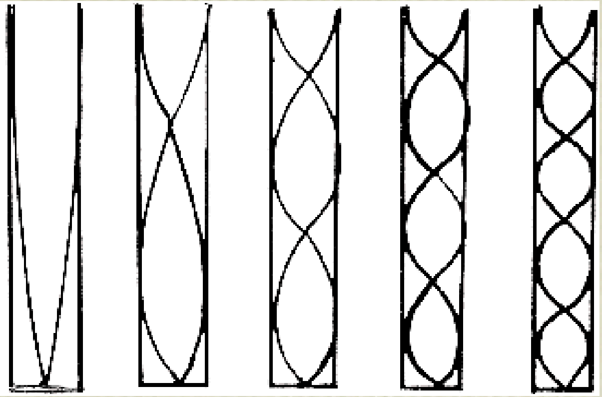
What Physical Modelling is NOT

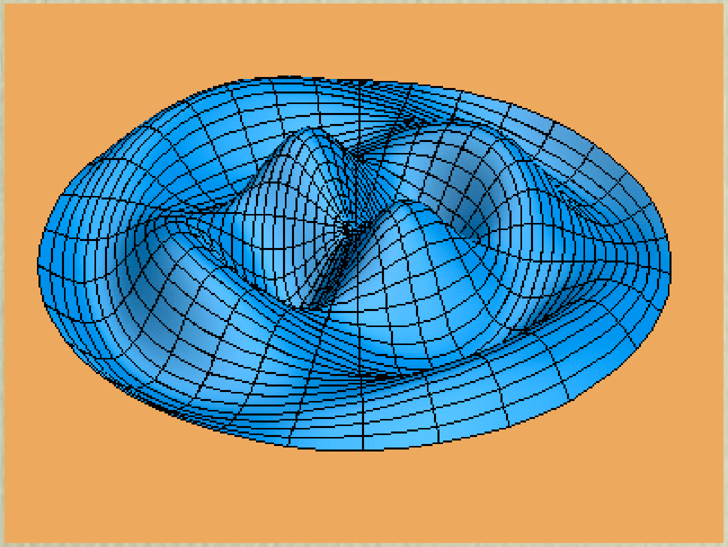
- NOT synthesis by *ad hoc* similarity of sound (e.g. subtractive synthesis)

What IS Physical Modelling?

- computer sound synthesis based on physical theory of the vibrating object - string, reed & air column, membrane

modes of vibration for a cylindrical air column



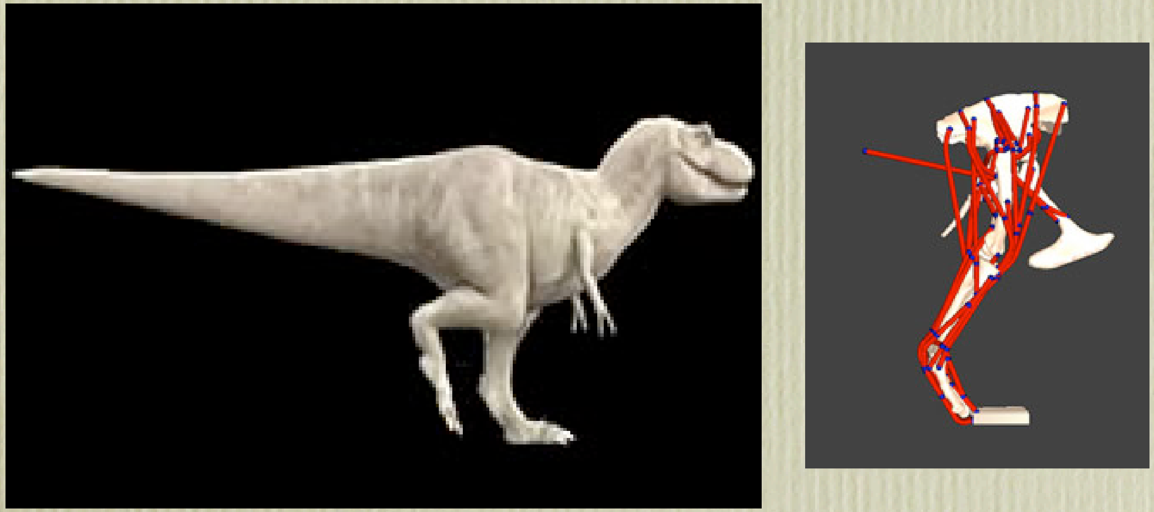


computer model of vibrating drum head

What IS Physical Modelling?

- analogous to computer graphics modelling of physical structures to simulate realistic dynamical behaviour

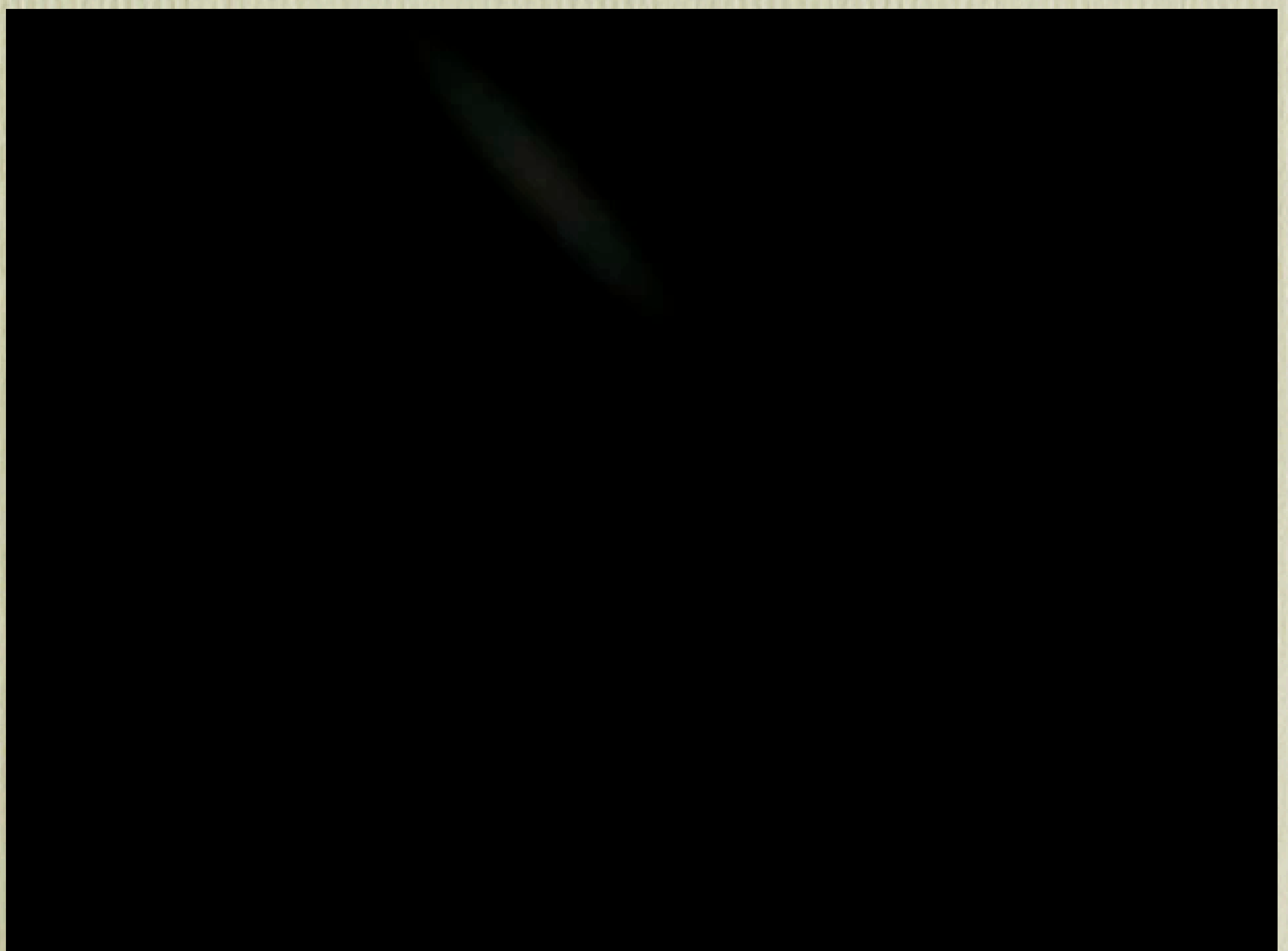
smoke synthesized with physical modelling (particle system)



animated dinosaur moves realistically by careful simulation of physical structure

What IS Physical Modelling?

- colliding galaxies synthesized by John Dubinski, U. of T. Dept. of Astronomy & Astrophysics



colliding galaxies

What IS Physical Modelling?

- simulates instrumental dynamics (i.e. behaviour)

both desirable behaviour....

What IS Physical Modelling?

- simulates instrumental dynamics (i.e. behaviour)

both desirable behaviour....

Violin physical model, staccato

What IS Physical Modelling?

- simulates instrumental dynamics (i.e. behavior)

and undesirable behaviour....

What IS Physical Modelling?

- simulates instrumental dynamics (i.e. behaviour)

and undesirable behaviour....

Violin physical model, bowing overpressure

The Problem with Physical Modelling...

from Julius O. Smith, "Physical Audio Signal Processing" 2006 http://ccrma.stanford.edu/

A More Complete Derivation of the String Wave Equation

Consider an elastic string under tension which is at rest along the x dimension. Let $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ denote the unit vectors in the x, y, and z directions, respectively. When a wave is present, a point $\mathbf{p} = (x, 0, 0)$ originally at x along the string is displaced to some point $\mathbf{a} = \mathbf{p} + d\mathbf{p}$ specified by the displacement vector

$$d\mathbf{p} = \mathbf{i}\xi + \mathbf{j}\eta + \mathbf{k}\zeta.$$

Note that typical derivations of the wave equation consider only the displacement η in the y direction. This more general treatment is adapted from [118].

The displacement of a neighboring point originally at $\mathbf{q} = (x + dx, 0, 0)$ along the string can be specified as

$$d\mathbf{q} = \mathbf{i}(\xi + d\xi) + \mathbf{j}(\eta + d\eta) + \mathbf{k}(\zeta + d\zeta).$$

Let K denote string tension along x when the string is at rest, and $\mathbf{K}$ denote the vector tension at the point $\mathbf{p}$ in the present displaced scenario under analysis. The net vector force acting on the infinitesimal string element between points $\mathbf{p}$ and $\mathbf{q}$ is given by the vector sum of the force $-\mathbf{K}$ at $\mathbf{p}$ and the force $\mathbf{K} + (\partial\mathbf{K}/\partial x)dx$ at $\mathbf{q}$, that is, $(\partial\mathbf{K}/\partial x)dx$. If the string has stiffness, the two forces will in general not be tangent to the string at these points. The mass of the infinitesimal string element is $\epsilon\, dx$, where ϵ denotes the mass per unit length of the string at rest. Applying Newton's second law gives

$$\frac{\partial\mathbf{K}}{\partial x} = \epsilon\frac{\partial^2\mathbf{p}}{\partial t^2} \qquad \text{(F.1)}$$

where dx has been canceled on both sides of the equation. Note that no approximations have been made so far.

The next step is to express the force $\mathbf{K}$ in terms of the tension K of the string at rest, the elastic constant of the string, and geometrical factors. The displaced string element $\mathbf{pq}$ is the vector

$$d\mathrm{s} = \mathbf{i}(dx + d\xi) + \mathbf{j}d\eta + \mathbf{k}d\zeta \qquad \text{(F.2)}$$

$$= \left[\mathbf{i}\left(1 + \frac{\partial\xi}{\partial x}\right) + \mathbf{j}\frac{\partial\eta}{\partial x} + \mathbf{k}\frac{\partial\zeta}{\partial x}\right] dx \qquad \text{(F.3)}$$

having magnitude

$$ds = \sqrt{\left(1 + \frac{\partial\xi}{\partial x}\right)^2 + \left(\frac{\partial\eta}{\partial x}\right)^2 + \left(\frac{\partial\zeta}{\partial x}\right)^2}. \qquad \text{(F.4)}$$

Karplus-Strong Synthesis

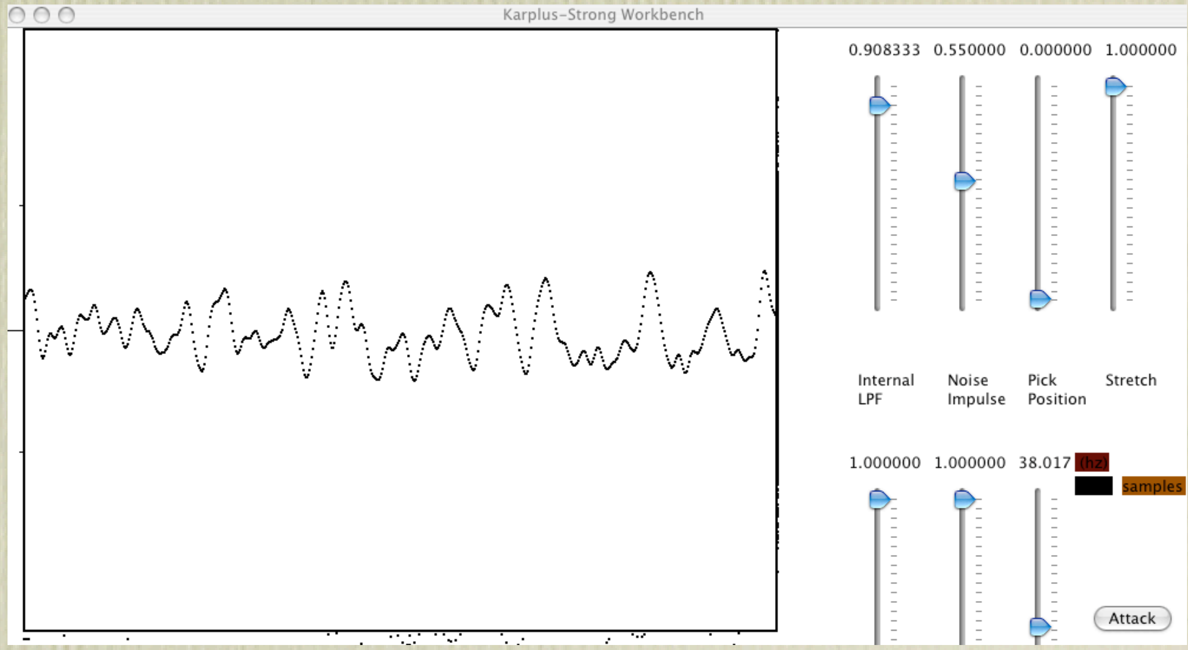
- named after Stanford grad. students Kevin Karplus and Alex Strong
  - Kevin Karplus, Alex Strong (1983). "Digital Synthesis of Plucked String and Drum Timbres". *Computer Music Journal* **7** (2): 43-55.
- efficient method for 8-bit microprocessor
- fill wavetable with random numbers
- average successive samples each time through the loop
  - averaging amounts to:
    - low-pass filtering (frequency domain description)
    - waveform smoothing (time domain description)

MidiForth
Karplus Workbench

Steady State Random Noise

Internal Low Pass Filter (LPF)

Varying Pitch



Karplus-Strong Synthesis

- Bruno Degazio - *HeatNoise* (1987)

HeatNoise is a fantasy fantasy on the inter-relationship of signal and noise, meaning and error, chaos and order. Noise - taken broadly and metaphorically as the absence of meaning - and the emergence of meaning from noise is presented with sounds synthesized by means of the same fractal process used to generate the structure of the work; with the noisy sounds of speech, the sibilants, plosives and fricatives without which language would be unintelligible; with radio transmissions, including Neil Armstrong's famous non sequitur at the first moon landing; and with sounds, musical and otherwise, that employ noise in various ways to communicate a message.

Out of the opening chaos through the progressively greater disturbances of the underlying order, noise overwhelms meaning until we arrive at the place where the lost messages end - the radio transmissions that were never received, the cries for help that were never heard, the final gasps of those who died alone... Curiously enough, just as researchers in information theory found that indelicate four letter words were the first to emerge from the chaos of random letter orderings, so here we discover that the last sound to be heard as the chaos engulfs us is not profanity but... rock music.

HeatNoise is one of a series of algorithmic compositions applying principles of fractal geometry to music. The structural foundation for the work is an extended rhythmic figure generated by the fractal equation used to describe errors due to thermal noise encountered in data transmission.

Karplus-Strong Synthesis

- Bruno Degazio - *HeatNoise* (1987)

HeatNoise is a fantasy on the inter-relationship of signal and noise, meaning and error, chaos and order. Noise - taken broadly and metaphorically as the absence of meaning - and the emergence of meaning from noise is presented with sounds synthesized by means of the same fractal process used to generate the structure of the work; with the noisy sounds of speech, the sibilants, plosives and fricatives without which language would be unintelligible; with radio transmissions, including Neil Armstrong's famous non sequitur at the first moon landing; and with sounds, musical and otherwise, that employ noise in various ways to communicate a message.

Out of the opening chaos through the progressively greater disturbances of the underlying order, noise overwhelms meaning until we arrive at the place where the lost messages end - the radio transmissions that were never received, the cries for help that were never heard, the final gasps of those who died alone... Curiously enough, just as researchers in information theory found that indelicate four letter words were the first to emerge from the chaos of random letter orderings, so here we discover that the last sound to be heard as the chaos engulfs us is not profanity but... rock music.

HeatNoise is one of a series of algorithmic compositions applying principles of fractal geometry to music. The structural foundation for the work is an extended rhythmic figure generated by the fractal equation used to describe errors due to thermal noise encountered in data transmission.

Waveguide Synthesis

- Stanford Prof. Julius O. Smith realized that this looped wavetable was equivalent to a digital representation of a vibrating string (or air column).
- developed the theoretical basis for what later became Waveguide Synthesis
- patented by Stanford in 1989 and licensed to Yamaha in 1994
- Yamaha's previous licensing relationship with Stanford included *FM Synthesis*, which resulted in the best selling synthesizer of the period, (Yamaha DX7) and the 2nd most lucrative licensing agreement in Stanford's history ($20,000,000)

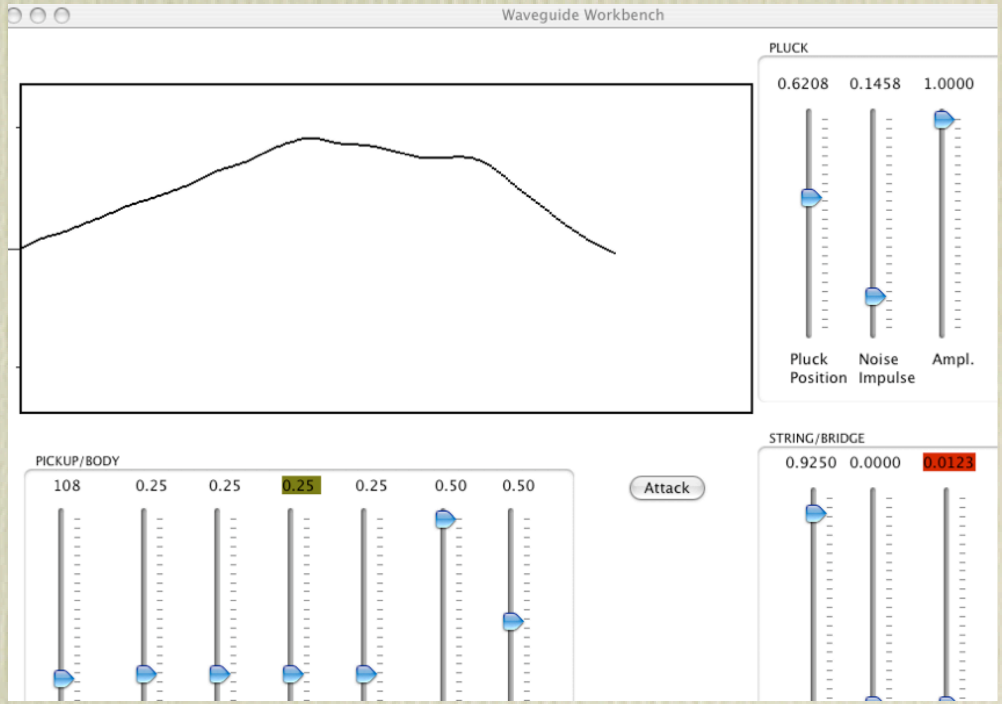
MidiForth
Waveguide Workbench

Pluck Position

Bridge LPF

Varying Pickup Position

Varying Pitch



JOS Proposed Clarinet Model (1986)

1.1. The Clarinet

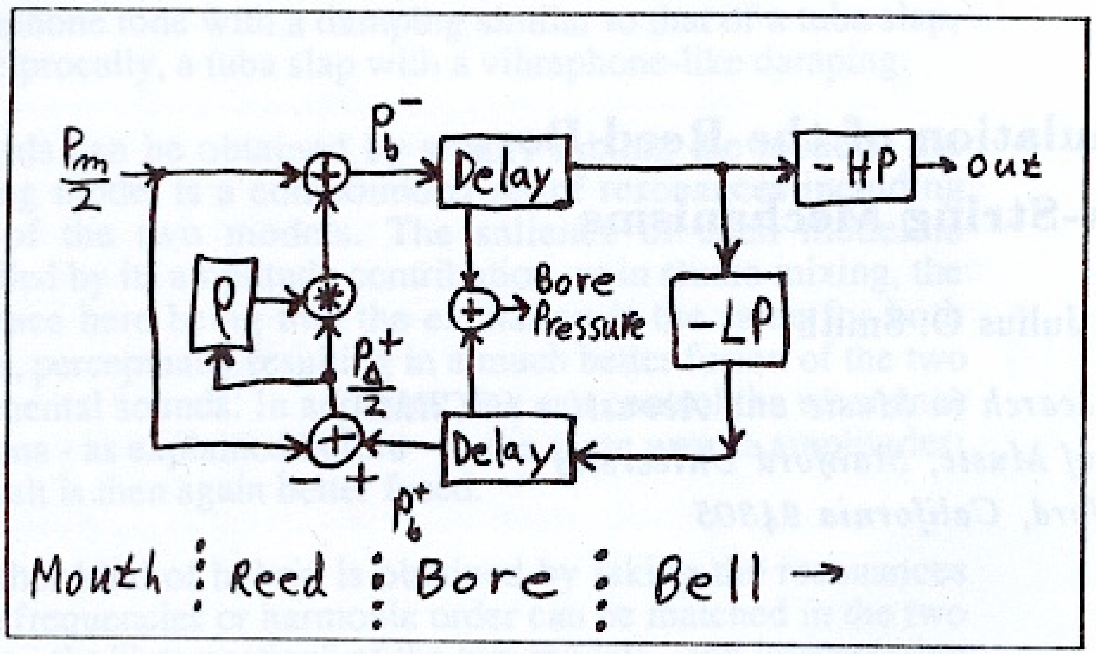
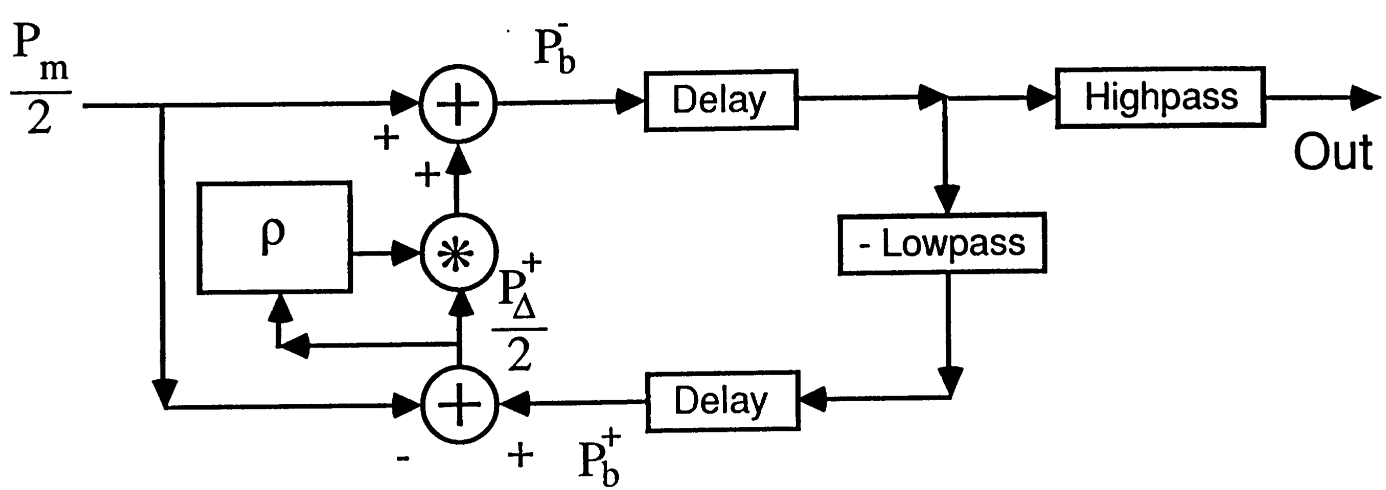


Figure 1. Model of a single-reed, cylindrical-bore woodwind.

from "*Efficient Simulation of the Reed-Bore and Bow-String Mechanisms*", Proceedings of the International Computer Music Conference, The Hague, 1986

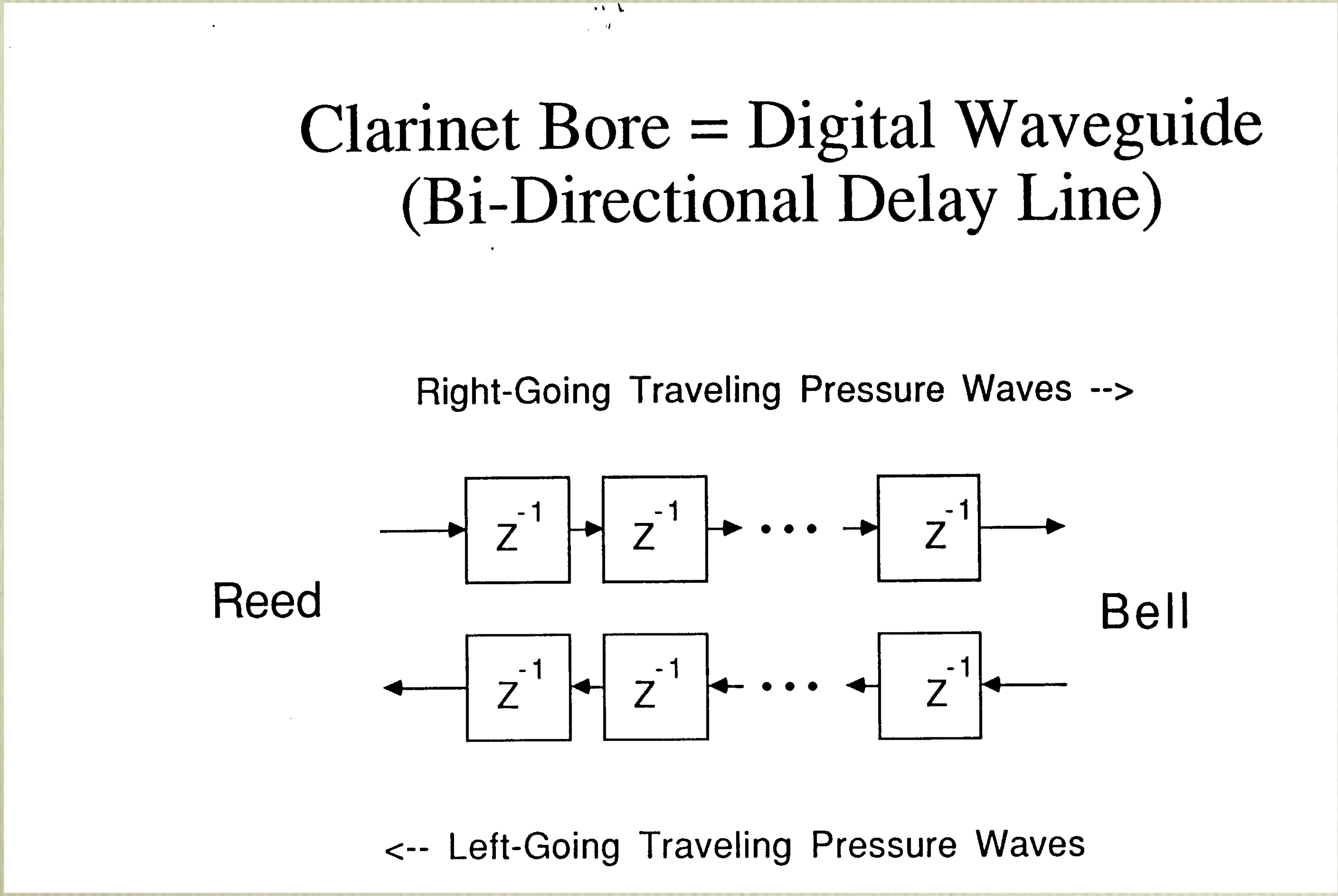
JOS Proposed Clarinet Model (1991)

Proposed Clarinet Implementation

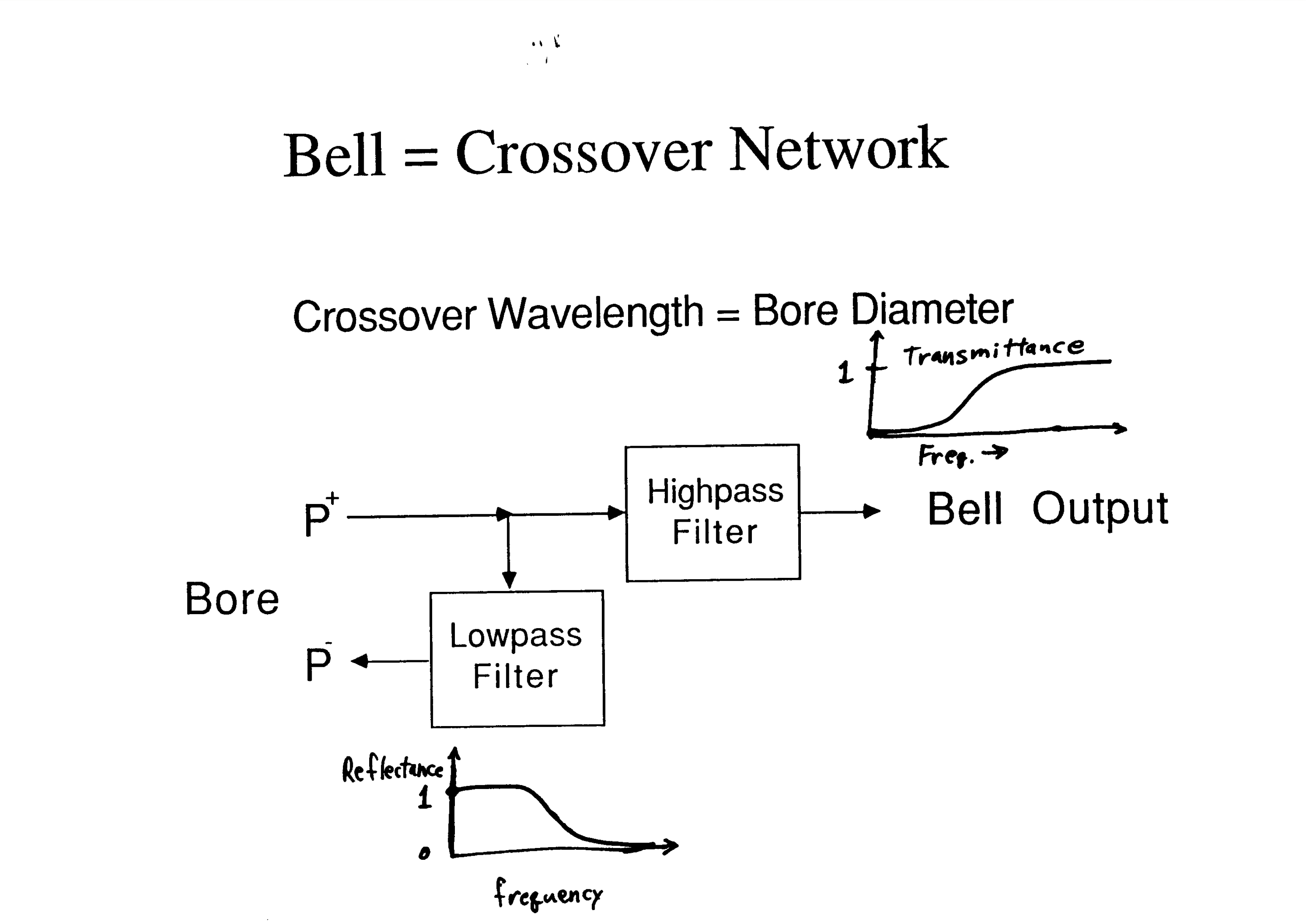


$$P_b^- = \rho\left(\frac{P_\Delta^+}{2}\right)\frac{P_\Delta^+}{2} + \frac{P_m}{2}$$

JOS Proposed Clarinet Model (1991)



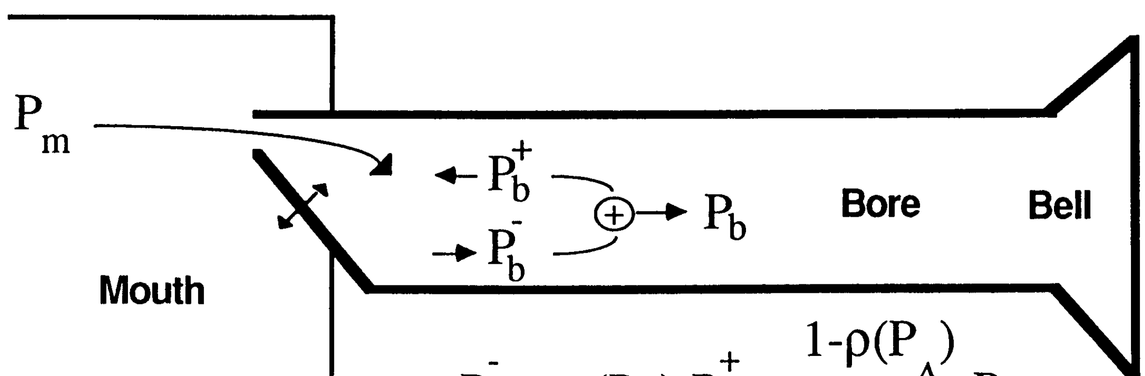
JOS Proposed Clarinet Model (1991)



JOS Proposed Clarinet Model (1991)

Reed = Pressure-Controlled Valve

Bore sees a time-varying reflection-coefficient
plus a time-varying mouth-pressure input



$$P_b^- = \rho(P_\Delta)\, P_b^+ + \frac{1-\rho(P_\Delta)}{2} P_m$$

$$P_\Delta = P_b - P_m$$

$$P_b = P_b^+ + P_b^-$$

JOS Proposed Violin Model (1986)

1.2. The Bowed String

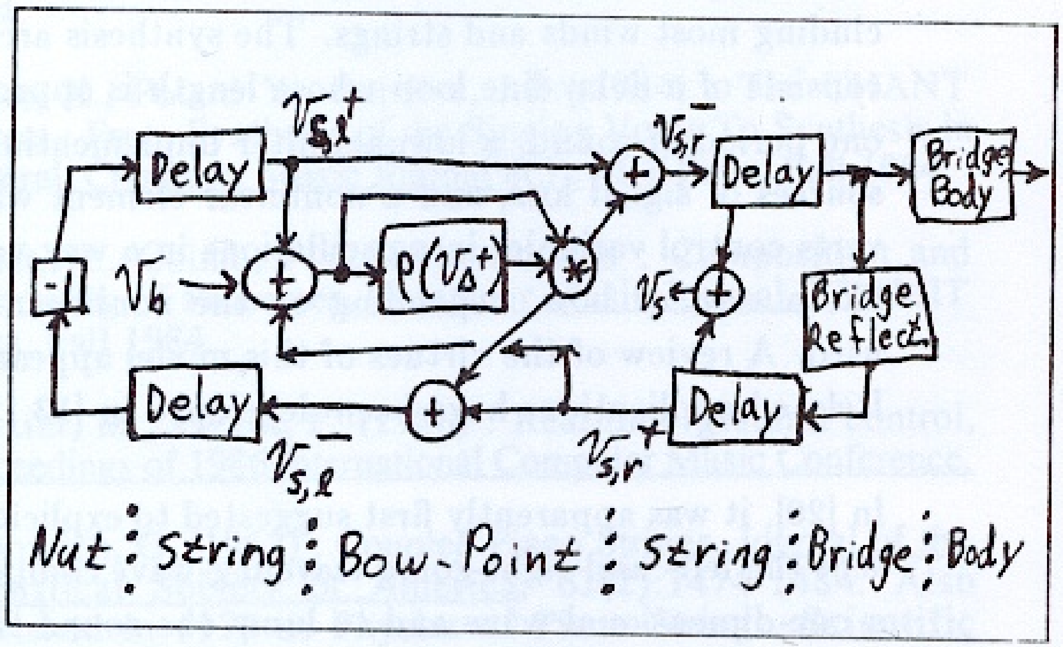


Figure 2. Basic model for a bowed string.

from "*Efficient Simulation of the Reed-Bore and Bow-String Mechanisms*", Proceedings of the International Computer Music Conference, The Hague, 1986

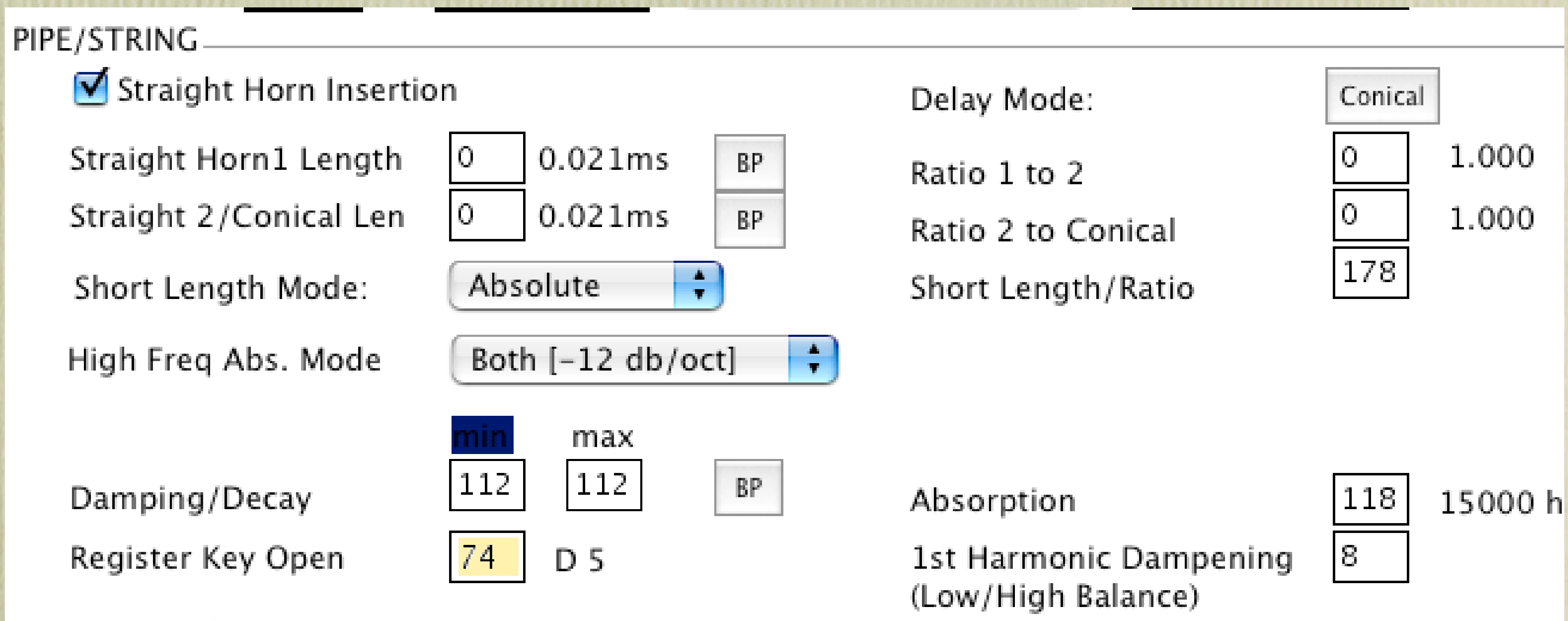
Yamaha VL1 Synthesizer

- Julius Smith & Stanford U. patented these techniques in late 1980s
- Yamaha licensed the patent in early '90s
- Vl1 synthesizer was the first product of Yamaha's license of Stanford's waveguide technology
- 2 voices, 48 khz, 16 bit
- optimized for simulation of woodwind and brass instruments, esp. saxophones

VL1 Architecture 1

	Driver	*Resonator*	*Modifier*
WW	reed	Pipe	Bell
Brass	lips	Pipe	Bell
String	bow	String	Body

VL1 Pipe Parameters



VL1 - Tube Shape

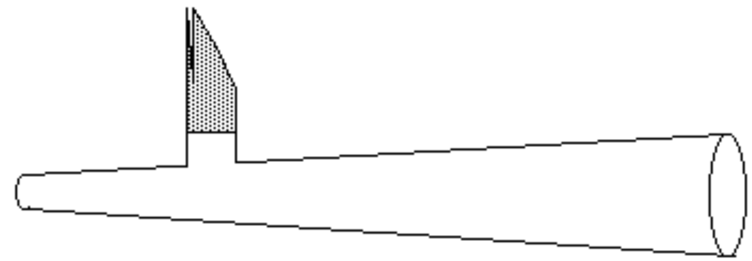


FIGURE 2: Absorption High. Cone tapered, minimum flare.

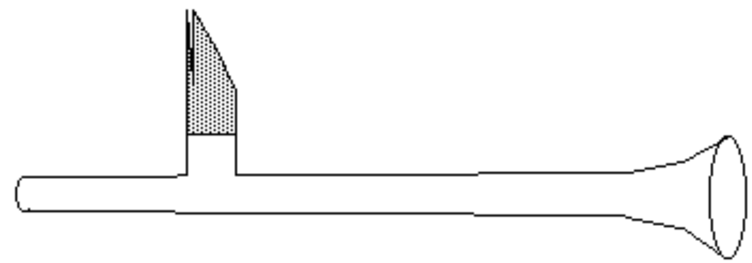


FIGURE 3: Absorption Low. Cone nearly straight, greater flaring.

Effect of Absorption on shape of Tube & Bell

Imitative Synthesis 1 - Clarinet

Mozart – Clarinet Quintet

prg. 033 ClasClarBD

Imitative Synthesis 1 - Clarinet

Mozart – Clarinet Quintet

prg. 033 ClasClarBD

Imitative Synthesis 1 - Clarinet

prg. 033 ClasClarBD

Instrumental Behaviour:

- note transitions: legato vs tongued
- harmonic overblowing

Imitative Synthesis 2 - Oboe

prg. 034 Oboe2md

- J.S. Bach BWV 1060 - Double Concerto

-all instruments are physically modelled except harpsichord

Imitative Synthesis 2 - Oboe

prg. 034 Oboe2md

- J.S. Bach BWV 1060 - Double Concerto

-all instruments are physically modelled except harpsichord

Imitative Synthesis 2 - French Horn

prg 038

Imitative Synthesis 2 - French Horn

prg 037

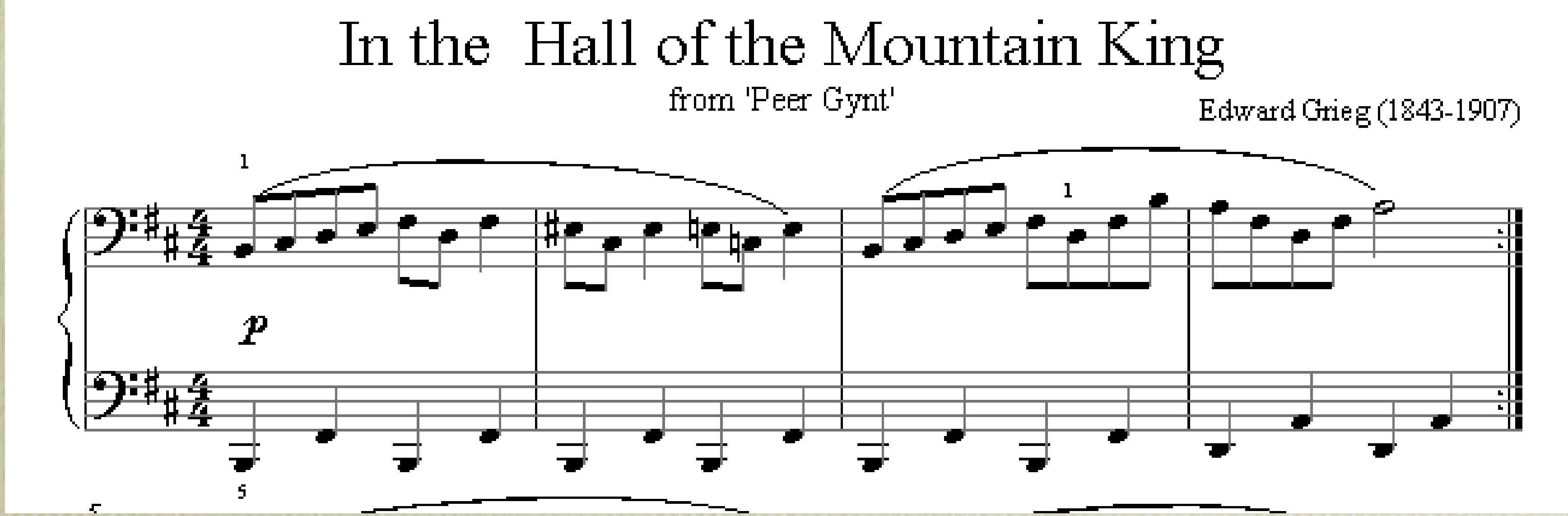
Instrumental Behavior:

The Harmonic Series

Imitative Synthesis 2 - Bassoon

prg 005 - Bassoon

Instrumental Behaviour: Staccato - note onsets and endings



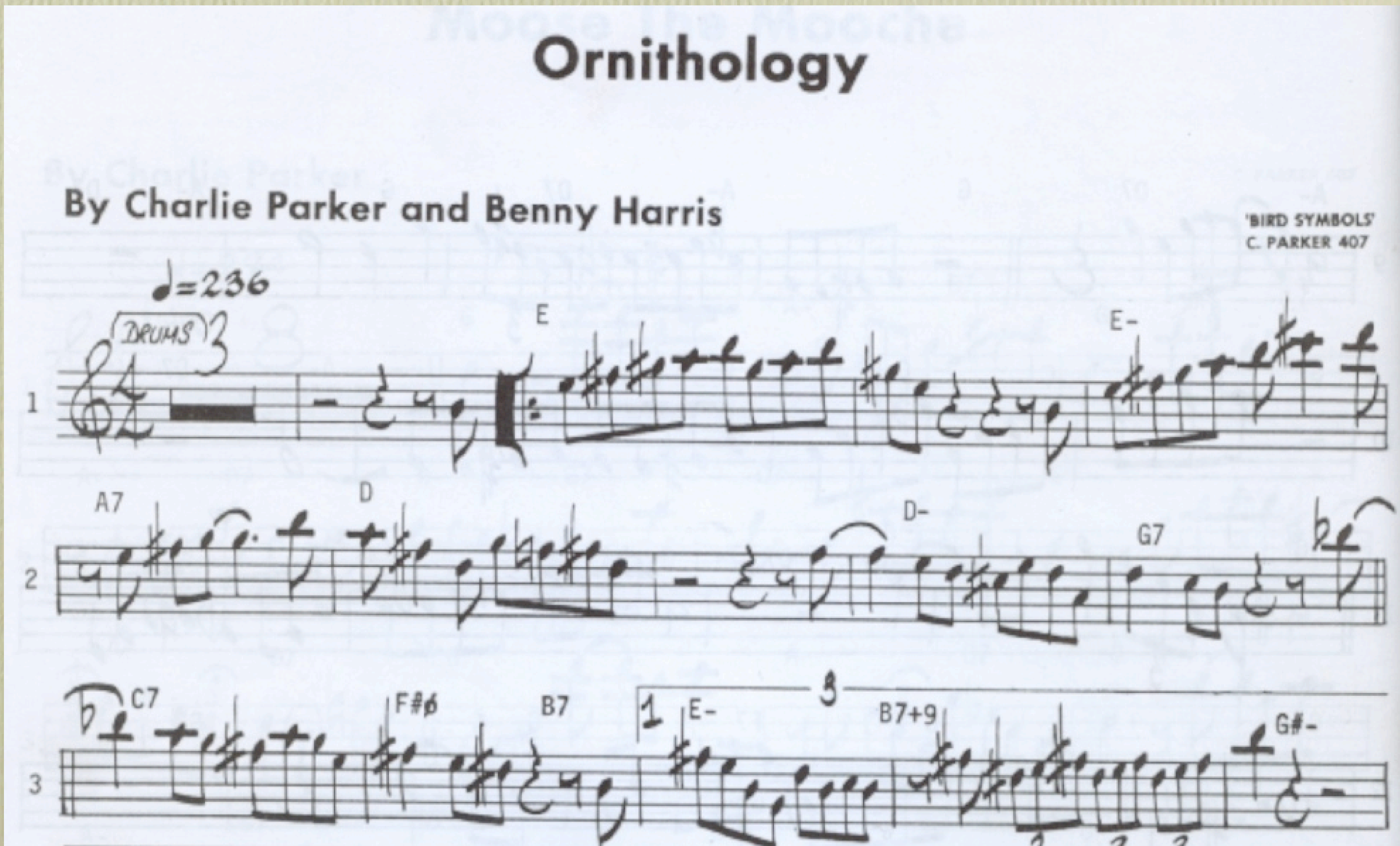
Imitative Synthesis 2 - Saxophone

prg 035 - Desmond

Instrumental Behaviour:
Acoustic Detail - breath noise

Imitative Synthesis 2 - Saxophone

prg 035 - Desmond



Emulative Synthesis 2 - Trombone

from *Ravel - Bolero*

Imitative Synthesis 1 - Woodwind Quintet

- Malcolm Arnold - Sea Shanty #1, arranged for "Virtual" Woodwind Quintet

Imitative Synthesis 1 - Woodwind Quintet

- Malcolm Arnold - Sea Shanty #1, arranged for "Virtual" Woodwind Quintet

  - Flute
  - Oboe
  - Clarinet
  - French Horn
  - Bassoon

Imitative Synthesis Plucked Strings

Instrumental Behaviour - Spanish Guitar - pluck vs gliss.

Conclusion

Bruno Degazio: *Algorithmic Animal Jive* (2003)

- all instruments (except piano) are physically modelled
  - plucked - bass guitar, violin pizz
  - struck - hand drum
  - wind - soprano sax, scat "voice"

- musical structure is algorithmically generated by a synthetic process based on analytical theories of Heinrich Schenker

Conclusion

Bruno Degazio: *Algorithmic Animal Jive* (2003)

- all instruments (except piano) are physically modelled
  - plucked - bass guitar, violin pizz
  - struck - hand drum
  - wind - soprano sax, scat "voice"

- musical structure is algorithmically generated by a synthetic process based on analytical theories of Heinrich Schenker

Why Physical Modelling?

- **Acoustically Detailed**
  - breath noise
  - harmonic series
  - onset transients & note transitions

- **Synthesis Parameters have real-world counterparts**
  - can be extended beyond real-world limits to explore "impossible" instruments

- **Playability**
  - responsive
  - expressive
  - realistic
  - unpredictable

Why Physical Modelling?

- **Elegance** - sound generated from first principles rather than through *ad hoc* accumulation of imitative features

$$
\begin{aligned}
d\mathbf{s} &= \mathbf{i}(dx + d\xi) + \mathbf{j}d\eta + \mathbf{k}d\zeta \\
&= \left[\mathbf{i}\left(1 + \frac{\partial \xi}{\partial x}\right) + \mathbf{j}\frac{\partial \eta}{\partial x} + \mathbf{k}\frac{\partial \zeta}{\partial x}\right] dx
\end{aligned}
$$

Why Physical Modelling?

- **Elegance** - sound generated from first principles rather than through *ad hoc* accumulation of imitative features

$$
\begin{aligned}
d\mathrm{s} &= \mathbf{i}(dx + d\xi) + \mathbf{j}d\eta + \mathbf{k}d\zeta \\
&= \left[\mathbf{i}\left(1 + \frac{\partial \xi}{\partial x}\right) + \mathbf{j}\frac{\partial \eta}{\partial x} + \mathbf{k}\frac{\partial \zeta}{\partial x}\right] dx
\end{aligned}
$$

- invokes a mystery - "*The Unreasonable Efficacy of Mathematics in Explaining the Physical World*" (Eugene Wigner, 1960, quantum physicist, Nobel prize winner)